Assumption: Let us assume $\sqrt{3}$ is rational number

- ⇨ $\sqrt{3} = \frac{a}{b}$,

Fact: where a and b are co – prime integers (common factor is only 1)

Cross multiply

$\sqrt{3}b = a$

Squaring on both sides

$3b^2 = a^2$ --- 1 eq

$b^2 = \frac{a^2}{3}$

As per theorem" Let p be a prime number. If p divides a^2, then p divides a"

As 3 divides a^2, then 3 divides a

- ⇨ 3 is a factor of a
- ⇨ 3c = a ---- 2 eq

substitute 2eq in 1eq

$3b^2 = (3c)^2$

$3b^2 = 9c^2$

$b^2 = 3c^2$

$$c^2 = \frac{b^2}{3}$$

As per theorem" Let p be a prime number. If p divides a^2, then p divides a"

As 3 divides b^2, then 3 divides b

- ⇨ 3 is a factor of b

Therefore, a and b have at least 3 as a common factor.

But this contradicts the fact that a and b are coprime.

This contradiction has arisen because of our incorrect assumption that √3 is rational.

So, we conclude that √3 is irrational

Show that $5 - \sqrt{3}$ is irrational.

Assumption: Let us assume that $5 - \sqrt{3}$ is rational.

Fact: $\sqrt{3}$ is irrational number

$5 - \sqrt{3} = a/b$, where a and b are co-prime integers

$5 - (a/b) = \sqrt{3}$

$$\frac{5b-a}{b} = \sqrt{3}$$

Since a and b are integers, we get $\frac{5b-a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.

So, we conclude that $5 - \sqrt{3}$ – is irrational.